

CONFIDENCE INTERVAL



A die (*singular of dice*) has 6 sides

Probability of rolling a **5** is
1 / 6 or **17%**



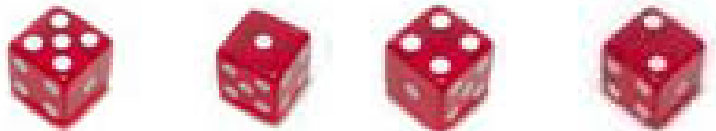
Probability of a **5** OR a **1** is
2 / 6, or **33%**



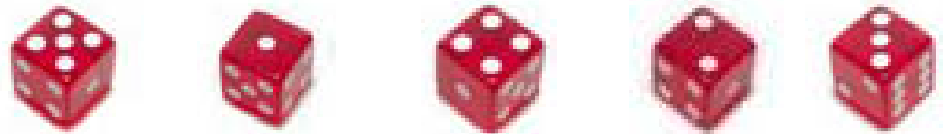
Prob of **5** or **1** or **4** is
3 / 6, or **50%**



5 or **1** or **4** or **2** = **66%**



$5/6 = 83\%$



$6/6 = 100\%$



No. of Choices	Confidence Level
1	17%
2	33%
3	50%
4	67%
5	83%
6	100%

Confidence Interval

SAT Scores --- $\mu = 500$, $\sigma = 100$

Assume a sample:

$$n = 225 \quad \bar{x} = 606 \quad (N = 10,000 \quad \mu = ?)$$

$$\text{C.I.} = \text{Sample Mean} \pm Z \times (?)$$

where $Z = z$ value for selected confidence level

(usually 95% or 99%)

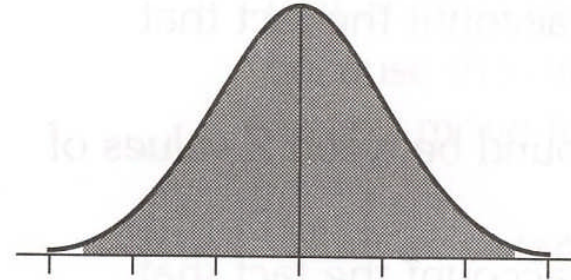
and $(?)$ is the Standard Error of the Mean

$$\sigma = 100 \quad n = 225 \quad x = 606$$

Confidence Level = 99%

Half above, half below the mean

$\frac{1}{2}$ of 99% is **0.4950**



Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357
0.03	0.0120	0.53	0.2019	1.03	0.3485	1.53	0.4370
0.04	0.0160	0.54	0.2054	1.04	0.3508	1.54	0.4382
0.05	0.0199	0.55	0.2088	1.05	0.3531	1.55	0.4394
0.06	0.0239	0.56	0.2123	1.06	0.3554	1.56	0.4406
0.07	0.0279	0.57	0.2157	1.07	0.3577	1.57	0.4418
0.08	0.0319	0.58	0.2190	1.08	0.3599	1.58	0.4429
0.09	0.0359	0.59	0.2224	1.09	0.3621	1.59	0.4441
0.10	0.0398	0.60	0.2257	1.10	0.3643	1.60	0.4452
0.11	0.0438	0.61	0.2291	1.11	0.3665	1.61	0.4463
0.12	0.0478	0.62	0.2324	1.12	0.3686	1.62	0.4474
0.13	0.0517	0.63	0.2357	1.13	0.3708	1.63	0.4484
0.14	0.0557	0.64	0.2389	1.14	0.3729	1.64	0.4495
0.15	0.0596	0.65	0.2422	1.15	0.3749	1.65	0.4505
0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525
0.18	0.0714	0.68	0.2517	1.18	0.3810	1.68	0.4535
0.19	0.0753	0.69	0.2549	1.19	0.3830	1.69	0.4545
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554
0.21	0.0832	0.71	0.2611	1.21	0.3869	1.71	0.4564
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664
0.34	0.1331	0.84	0.2995	1.34	0.4099	1.84	0.4671
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686
0.37	0.1443	0.87	0.3078	1.37	0.4147	1.87	0.4693
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4699
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713
0.41	0.1591	0.91	0.3186	1.41	0.4207	1.91	0.4719
0.42	0.1628	0.92	0.3212	1.42	0.4222	1.92	0.4726
0.43	0.1664	0.93	0.3238	1.43	0.4236	1.93	0.4732
0.44	0.1700	0.94	0.3264	1.44	0.4251	1.94	0.4738
0.45	0.1736	0.95	0.3289	1.45	0.4265	1.95	0.4744
0.46	0.1772	0.96	0.3315	1.46	0.4279	1.96	0.4750
0.47	0.1808	0.97	0.3340	1.47	0.4292	1.97	0.4756
0.48	0.1844	0.98	0.3365	1.48	0.4306	1.98	0.4761
0.49	0.1879	0.99	0.3389	1.49	0.4319	1.99	0.4767

Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z
2.00	0.4772	2.50	0.4938	3.00	0.4987	3.50	0.4998
2.01	0.4778	2.51	0.4940	3.01	0.4987	3.60	0.4998
2.02	0.4783	2.52	0.4941	3.02	0.4987	3.70	0.4999
2.03	0.4788	2.53	0.4943	3.03	0.4988	3.80	0.4999
2.04	0.4793	2.54	0.4945	3.04	0.4988	3.90	0.4999
2.05	0.4798	2.55	0.4946	3.05	0.4989		
2.06	0.4803	2.56	0.4948	3.06	0.4989		
2.07	0.4808	2.57	0.4949	3.07	0.4989		
2.08	0.4812	2.58	0.4951	3.08	0.4990		
2.09	0.4817	2.59	0.4952	3.09	0.4990		
2.10	0.4821	2.60	0.4953	3.10	0.4990		
2.11	0.4826	2.61	0.4955	3.11	0.4991		
2.12	0.4830	2.62	0.4956	3.12	0.4991		
2.13	0.4834	2.63	0.4957	3.13	0.4991		
2.14	0.4838	2.64	0.4959	3.14	0.4992		
2.15	0.4842	2.65	0.4960	3.15	0.4992		
2.16	0.4846	2.66	0.4961	3.16	0.4992		
2.17	0.4850	2.67	0.4962	3.17	0.4992		
2.18	0.4854	2.68	0.4963	3.18	0.4993		
2.19	0.4857	2.69	0.4964	3.19	0.4993		
2.20	0.4861	2.70	0.4965	3.20	0.4993		
2.21	0.4864	2.71	0.4966	3.21	0.4993		
2.22	0.4868	2.72	0.4967	3.22	0.4994		
2.23	0.4871	2.73	0.4968	3.23	0.4994		
2.24	0.4875	2.74	0.4969	3.24	0.4994		
2.25	0.4878	2.75	0.4970	3.25	0.4994		
2.26	0.4881	2.76	0.4971	3.26	0.4994		
2.27	0.4884	2.77	0.4972	3.27	0.4995		
2.28	0.4887	2.78	0.4973	3.28	0.4995		
2.29	0.4890	2.79	0.4974	3.29	0.4995		
2.30	0.4893	2.80	0.4974	3.30	0.4995		
2.31	0.4896	2.81	0.4975	3.31	0.4995		
2.32	0.4898	2.82	0.4976	3.32	0.4995		
2.33	0.4901	2.83	0.4977	3.33	0.4996		
2.34	0.4904	2.84	0.4977	3.34	0.4996		
2.35	0.4906	2.85	0.4978	3.35	0.4996		
2.36	0.4909	2.86	0.4979	3.36	0.4996		
2.37	0.4911	2.87	0.4979	3.37	0.4996		
2.38	0.4913	2.88	0.4980	3.38	0.4996		
2.39	0.4916	2.89	0.4981	3.39	0.4997		
2.40	0.4918	2.90	0.4981	3.40	0.4997		
2.41	0.4920	2.91	0.4982	3.41	0.4997		
2.42	0.4922	2.92	0.4982	3.42	0.4997		
2.43	0.4925	2.93	0.4983	3.43	0.4997		
2.44	0.4927	2.94	0.4984	3.44	0.4997		
2.45	0.4929	2.95	0.4984	3.45	0.4997		
2.46	0.4931	2.96	0.4985	3.46	0.4997		
2.47	0.4932	2.97	0.4985	3.47	0.4997		
2.48	0.4934	2.98	0.4986	3.48	0.4997		
2.49	0.4936	2.99	0.4986	3.49	0.4998		

Z=2.58

Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z
2.00	0.4772	2.50	0.4938	3.00	0.4987	3.50	0.4998
2.01	0.4778	2.51	0.4940	3.01	0.4987	3.60	0.4998
2.02	0.4783	2.52	0.4941	3.02	0.4987	3.70	0.4999
2.03	0.4788	2.53	0.4943	3.03	0.4988	3.80	0.4999
2.04	0.4793	2.54	0.4945	3.04	0.4988	3.90	0.4999
2.05	0.4798	2.55	0.4946	3.05	0.4989		
2.06	0.4803	2.56	0.4948				
2.07	0.4808	2.57	0.4949				
2.08	0.4812	2.58	0.4951				
2.09	0.4817	2.59	0.4952				
2.10	0.4821	2.60	0.4953				
2.11	0.4826	2.61	0.4955				
2.12	0.4830	2.62	0.4956				
2.13	0.4834	2.63	0.4957				
2.14	0.4838	2.64	0.4959				
2.15	0.4842	2.65	0.4960				
2.16	0.4846	2.66	0.4961	3.16	0.4992		
2.17	0.4850	2.67	0.4962	3.17	0.4992		
2.18	0.4854	2.68	0.4963	3.18	0.4993		
2.19	0.4857	2.69	0.4964	3.19	0.4993		
2.20	0.4861	2.70	0.4965	3.20	0.4993		
2.21	0.4864	2.71	0.4966	3.21	0.4993		
2.22	0.4868	2.72	0.4967	3.22	0.4994		
2.23	0.4871	2.73	0.4968	3.23	0.4994		
2.24	0.4875	2.74	0.4969	3.24	0.4994		
2.25	0.4878	2.75	0.4970	3.25	0.4994		
2.26	0.4881	2.76	0.4971	3.26	0.4994		
2.27	0.4884	2.77	0.4972	3.27	0.4995		
2.28	0.4887	2.78	0.4973	3.28	0.4995		
2.29	0.4890	2.79	0.4974	3.29	0.4995		
2.30	0.4893	2.80	0.4974	3.30	0.4995		
2.31	0.4896	2.81	0.4975	3.31	0.4995		
2.32	0.4898	2.82	0.4976	3.32	0.4995		
2.33	0.4901	2.83	0.4977	3.33	0.4996		
2.34	0.4904	2.84	0.4977	3.34	0.4996		

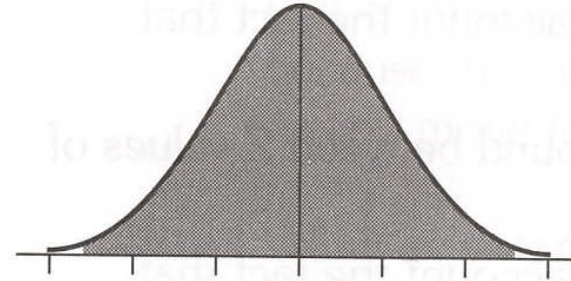
Half of 99% =
0.4950
Corresponds to
z = 2.58

$$\sigma = 100 \quad n = 225 \quad x = 606$$

Confidence Level = 99%

Half above, half below the mean

½ of 99% is **0.4950**



Z = 2.58 (for 99%)

$$\text{C.I.} = \text{Sample Mean} \pm \mathbf{Z} \times (\sigma / \sqrt{n})$$

$$\text{C.I.} = 606 \pm (\mathbf{2.58} \times (100 / \sqrt{225}))$$

$$\text{C.I.} = 606 \pm 17.21$$

$$\text{C.I.} = 606 \pm 17 \text{ (rounded to whole number)}$$

$$\text{C.I.} = \mathbf{589 \text{ to } 623}$$

Translation: 99 times out of 100 our results would contain the mean of the population

Confidence Interval: 588 – 623

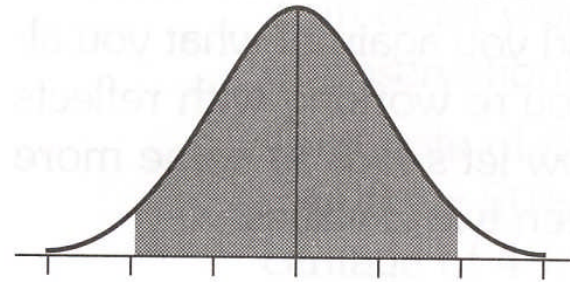
Confidence Level: 99%

$$\sigma = 100 \quad n = 225 \quad x = 606$$

Confidence Level = 95%

Half above, half below the mean

$\frac{1}{2}$ of 95% is **0.4750**



0.16	0.0636	0.66	0.2454	1.16	0.3770	1.66	0.4515
0.17	0.0675	0.67	0.2486	1.17	0.3790	1.67	0.4525
0.18	0.0714	0.68	0.2517	1.18	0.3810	1.68	0.4535
0.19	0.0753	0.69	0.2549	1.19	0.3830	1.69	0.4545
0.20	0.0793	0.70	0.2580	1.20	0.3849	1.70	0.4554
0.21	0.0832	0.71	0.2611	1.21	0.3869	1.71	0.4564
0.22	0.0871	0.72	0.2642	1.22	0.3888	1.72	0.4573
0.23	0.0910	0.73	0.2673	1.23	0.3907	1.73	0.4582
0.24	0.0948	0.74	0.2704	1.24	0.3925	1.74	0.4591
0.25	0.0987	0.75	0.2734	1.25	0.3944	1.75	0.4599
0.26	0.1026	0.76	0.2764	1.26	0.3962	1.76	0.4608
0.27	0.1064	0.77	0.2794	1.27	0.3980	1.77	0.4616
0.28	0.1103	0.78	0.2823	1.28	0.3997	1.78	0.4625
0.29	0.1141	0.79	0.2852	1.29	0.4015	1.79	0.4633
0.30	0.1179	0.80	0.2881	1.30	0.4032	1.80	0.4641
0.31	0.1217	0.81	0.2910	1.31	0.4049	1.81	0.4649
0.32	0.1255	0.82	0.2939	1.32	0.4066	1.82	0.4656
0.33	0.1293	0.83	0.2967	1.33	0.4082	1.83	0.4664
0.34	0.1331	0.84	0.2995	1.34	0.4099	1.84	0.4671
0.35	0.1368	0.85	0.3023	1.35	0.4115	1.85	0.4678
0.36	0.1406	0.86	0.3051	1.36	0.4131	1.86	0.4686
0.37	0.1443	0.87	0.3078	1.37	0.4147	1.87	0.4693
0.38	0.1480	0.88	0.3106	1.38	0.4162	1.88	0.4699
0.39	0.1517	0.89	0.3133	1.39	0.4177	1.89	0.4706
0.40	0.1554	0.90	0.3159	1.40	0.4192	1.90	0.4713
0.41	0.1591	0.91	0.3186			1.91	0.4719
0.42	0.1628	0.92	0.3212			1.92	0.4726
0.43	0.1664	0.93	0.3238			1.93	0.4732
0.44	0.1700	0.94	0.3264			1.94	0.4738
0.45	0.1736	0.95	0.3289			1.95	0.4744
0.46	0.1772	0.96	0.3315			1.96	0.4750
0.47	0.1808	0.97	0.3340			1.97	0.4756
0.48	0.1844	0.98	0.3365			1.98	0.4761
0.49	0.1879	0.99	0.3389			1.99	0.4767

Z =

Half of 95% = 0.4750

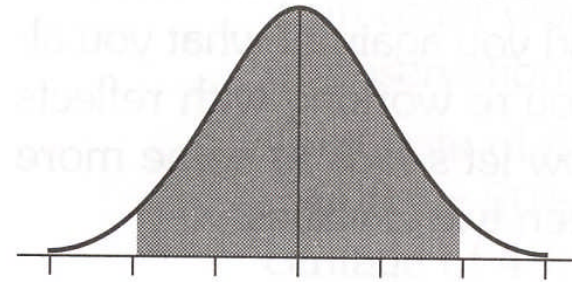
Corresponds to
z=1.96

$$\sigma = 100 \quad n = 225 \quad \bar{x} = 606$$

Confidence Level = 95%

Half above, half below the mean

½ of 95% is **0.4750**



Z = 1.96 (for 95%)

$$\text{C.I.} = \text{Sample Mean} \pm Z \times (\sigma / \sqrt{n})$$

$$\text{C.I.} = 606 \pm (1.96 \times (100 / \sqrt{225}))$$

$$\text{C.I.} = 606 \pm 1.96 * 6.67$$

$$\text{C.I.} = 606 \pm 13 \text{ (rounded to whole number)}$$

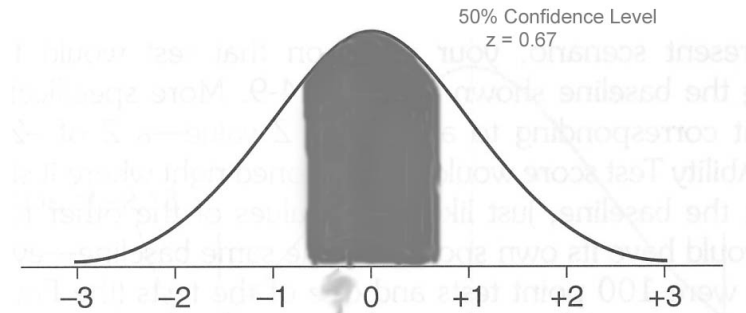
$$\text{C.I.} = \mathbf{593 \text{ to } 619}$$

$$\sigma = 100 \quad n = 225 \quad x = 606$$

Confidence Level = 50%

Half above, half below the mean

$\frac{1}{2}$ of 50% is **0.2500**



Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z	Z	Area Between Mean and Z
0.00	0.0000	0.50	0.1915	1.00	0.3413	1.50	0.4332
0.01	0.0040	0.51	0.1950	1.01	0.3438	1.51	0.4345
0.02	0.0080	0.52	0.1985	1.02	0.3461	1.52	0.4357
0.03	0.0120	0.53	0.2019	1.03	0.3483	1.53	0.4369
0.04	0.0160	0.54	0.2054	1.04	0.3504	1.54	0.4381
0.05	0.0199	0.55	0.2088	1.05	0.3525	1.55	0.4392
0.06	0.0239	0.56	0.2123	1.06	0.3544	1.56	0.4403
0.07	0.0279	0.57	0.2157	1.07	0.3563	1.57	0.4414
0.08	0.0319	0.58	0.2190	1.08	0.3581	1.58	0.4425
0.09	0.0359	0.59	0.2224	1.09	0.3599	1.59	0.4436
0.10	0.0398	0.60	0.2257	1.10	0.3617	1.60	0.4447
0.11	0.0438	0.61	0.2291	1.11	0.3634	1.61	0.4457
0.12	0.0478	0.62	0.2324	1.12	0.3651	1.62	0.4467
0.13	0.0517	0.63	0.2357	1.13	0.3667	1.63	0.4477
0.14	0.0557	0.64	0.2389	1.14	0.3683	1.64	0.4487
0.15	0.0596	0.65	0.2422	1.15	0.3699	1.65	0.4505
0.16	0.0636	0.66	0.2454	1.16	0.3715	1.66	0.4515
0.17	0.0675	0.67	0.2486	1.17	0.3730	1.67	0.4525
0.18	0.0714	0.68	0.2517	1.18	0.3745	1.68	0.4535
0.19	0.0753	0.69	0.2549	1.19	0.3760	1.69	0.4545
0.20	0.0792	0.70	0.2580	1.20	0.3774	1.70	0.4554
0.21	0.0831	0.71	0.2611	1.21	0.3788	1.71	0.4564
0.22	0.0870	0.72	0.2642	1.22	0.3802	1.72	0.4573
0.23	0.0909	0.73	0.2673	1.23	0.3815	1.73	0.4582
0.24	0.0947	0.74	0.2704	1.24	0.3828	1.74	0.4591
0.25	0.0985	0.75	0.2734	1.25	0.3841	1.75	0.4599
0.26	0.1023	0.76	0.2764	1.26	0.3854	1.76	0.4608
0.27	0.1064	0.77	0.2794	1.27	0.3866	1.77	0.4616
0.28	0.1103	0.78	0.2823	1.28	0.3878	1.78	0.4625
0.29	0.1141	0.79	0.2852	1.29	0.3890	1.79	0.4633
0.30	0.1179	0.80	0.2881	1.30	0.3901	1.80	0.4641
0.31	0.1217	0.81	0.2910	1.31	0.3912	1.81	0.4649
0.32	0.1255	0.82	0.2938	1.32	0.3922	1.82	0.4656

0.2500 is about halfway between these values (.2486 and .2517)

So z value is about halfway between 0.67 and 0.68 (=0.675)

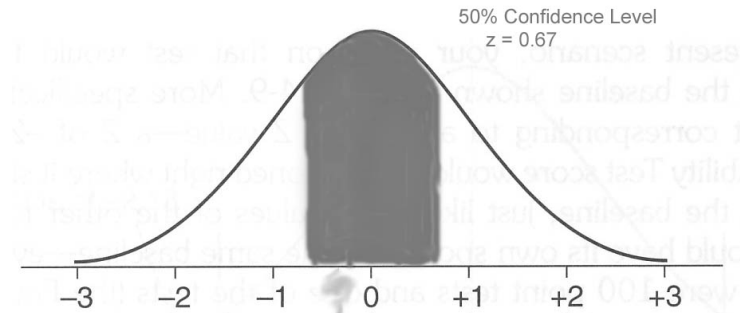
Z is between 0.67 & 0.68

$$\sigma = 100 \quad n = 225 \quad x = 606$$

Confidence Level = 50%

Half above, half below the mean

½ of 50% is **0.2500**



Z = 0.675 (for 50%)

$$\text{C.I.} = \text{Sample Mean} \pm \mathbf{Z} \times (\sigma / \sqrt{n})$$

$$\text{C.I.} = 606 \pm (\mathbf{0.675} \times (100 / \sqrt{225}))$$

$$\text{C.I.} = 606 \pm \mathbf{0.675} * 6.67$$

$$\text{C.I.} = 606 \pm \mathbf{5} \text{ (rounded to whole number)}$$

$$\text{C.I.} = \mathbf{601 \text{ to } 611}$$

Confidence Interval

Confidence Level

%	LL	UL	
99	589	- 623	34
95	593	- 619	26
68	599	- 613	14
50	601	- 611	10

